Realizing Hyperbolic Metamaterial

Personal Project

Advisors

STEFAN ENOCH
REDHA ABDEDDAIM

Prepared By
MOHAMMED ABDEDAIM
MD MAHBUB ALAM
MD SHOFIQUL ISLAM KHAN

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ABSTRACT

We began this project with an introduction to the very first metamaterial structures. From those literature we earned basic idea behind metamaterials and their wonderful properties. Then we started to explore further with a goal to realize hyperbolic metamaterial. In our journey towards that goal, we began with the concept of plasmons and metal structures response to the incident field which inspired us in structural engineering using metals. Eventually we proposed a very simple structure to realize hyperbolic metamaterial. We presented necessary arguments to support our proposal based on the knowledge we had earned earlier. Then we practically realized our proposed structure and did measurements in the lab which confirmed that we have successfully realized a hyperbolic metamaterial structure.

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Preamble

1.1 Negative Index

Consider light passing through a plate of glass. We know that light is an electromagnetic wave, consisting of oscillating electric and magnetic fields, and characterized by a wavelength λ. Because visible light has a wavelength that is hundreds of times larger than the atoms of which the glass is composed, the atomic details lose importance in describing how the glass interacts with light. In practice, we can average over the atomic scale, conceptually replacing the otherwise inhomogeneous medium by a homogeneous material characterized by just two macroscopic electromagnetic parameters: the electric permittivity ϵ, and the magnetic permeability, μ. Electrical Permittivity, ϵ and magnetic permeability, μ are concepts deeply embedded in electromagnetism and summaries the response of a homogeneous medium to electric and magnetic fields. These familiar quantities are usually thought of as positive numbers, but in principle negative values are allowed for both ϵ and μ and in practice negative values of . . . õ are realized at optical frequencies for metals. From ϵ and μ we can derive the refractive index,

$$n = \sqrt{\epsilon\mu} \tag{1.1}$$

which determines how light is bent on traversing from one material to another. ϵ, μ and n and are the three players in our story. The theme is what happens when these quantities take negative values either separately or together. The optical response of metals is determined by their electrons which are in essence a plasma of free charged particles. Their finite mass means that they do not respond instantaneously to electric fields but do so 90^0 out of phase. Hence the negative values of ϵ.

Naturally occurring materials do not have negative values of μ and artificial materials with negative μ appeared on the scene only recently. The trick was to produce a microstructured material containing coils of wire on a scale much smaller than the wavelength of radiation, constructed so that currents flow in response to a magnetic field. Careful design of these structures enables negative values of μ to be achieved though so far this has been possible only at microwave frequencies.

1.2 Metamaterials

These properties of negative permittivity or permeability or in a broad scale, negative refractive index are not natural properties of materials. They are assemblies of multiple individual elements fashioned from conventional materials such as metals or plastics, but the materials are usually constructed into repeating patterns, often with microscopic structures. Metamaterials derive their properties not from the compositional properties of the base materials, but from their exactingly-designed structures.

1.3 Plasmons

Plasmons can be described in the classical picture as an oscillation of free electron density with respect to the fixed positive ions in a metal. To visualize a plasma oscillation, imagine a cube of metal placed in an external electric field pointing to the right. Electrons will move to the left side (uncovering positive ions on the right side) until they cancel the field inside the metal. If the electric field is removed, the electrons move to the right, repelled by each other and attracted to the positive ions left bare on the right side. They oscillate back and forth at the plasma frequency until the energy is lost in some kind of resistance or damping. Plasmons are a quantization of this kind of oscillation. Plasma frequency defined [4, 7] as-

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m_{eff}} \tag{1.2}$$

where n is the number of electrons per unit volume, ϵ_0 permittivity of the vacuum, e is the charge of electron and m_{eff} is the effective mass of electron. Plasmons play a large role in the optical properties of metals. Light of frequencies below the plasma frequency is reflected, because the electrons in the metal screen the electric field of the light. Light of frequencies above the plasma frequency is transmitted, because the electrons cannot respond fast enough to screen it. In most metals, the plasma frequency is in the ultraviolet, making them shiny (reflective) in the visible range.

The plasmons have a profound impact on properties of metals, not least on their interaction with electromagnetic radiation where the plasmon produces a dielectric function of the form-

$$\epsilon(\omega) = \frac{\omega_p^2}{\omega(\omega + i\gamma)} \tag{1.3}$$

Here γ represents damping term representing dissipation of the plasmon‚Äôs energy into the system and ω is the frequency of the external field. This relation of permittivity and frequency is approximately independent of wave vector. In simple metals (aluminum) damping term γ is small compared to plasma frequency.

1.4 Why negative epsilon interesting?

Surface plasmons are coherent delocalized electron oscillations that exist at the interface between any two materials where the real part of the dielectric function changes sign across the interface (e.g. a metal-dielectric interface, such as a metal sheet in air). They have lower energy than bulk (or volume) plasmons.

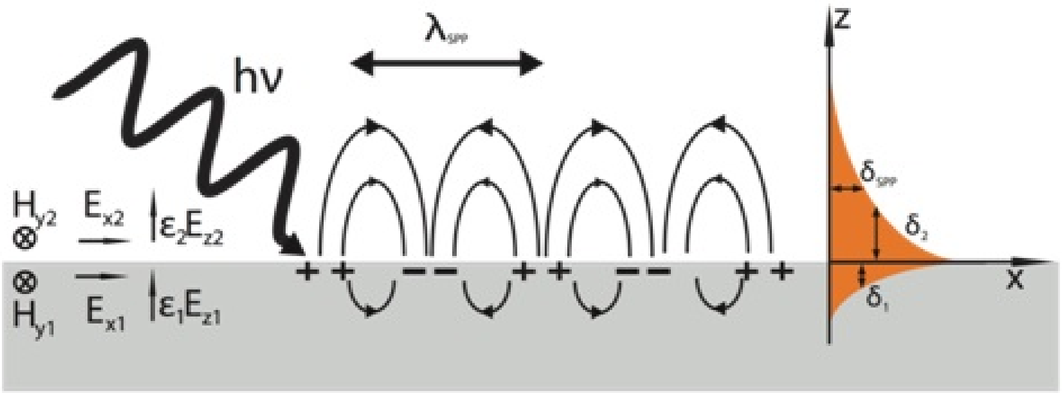


FIGURE 1.1. Schematic representation of an electron density wave propagating along a metal-dielectric interface.

The charge motion in a surface plasmon always creates electromagnetic fields outside (as well as inside) the metal. The total excitation, including both the charge motion and associated electromagnetic field, is called either a surface plasmon polariton at a planar interface, or a localized surface plasmon for the closed surface of a small particle.
The charge density oscillations and associated electromagnetic fields are called surface plasmon-polariton waves. Figure 1.1 illustrates its propagation along the metal-dielectric boundary, the intensity of the evanescent fields in each layer, and showing the direction and variables names for fields involved. The exponential dependence of the electromagnetic field intensity on the distance away from the interface is shown on the right.

Now if our metal is cut into two half then two surfaces created will be decorated with surface plasmons. This collective oscillation bound to the surface whose frequency is given by the following condition [4, 7],

$$\epsilon_1(\omega_s) + \epsilon_2(\omega_s) = 0 \tag{1.4}$$

Where ϵ_1 and ϵ_2 are the dielectric functions for material on either side of the interface. By choosing vacuum in one side and metal on the other gives. $\omega_s = \frac{\omega_p}{\sqrt{2}}$.

It is of course an essential precondition that ϵ for the metal be negative. Shape the metal into a sphere and another set of surface modes appears. Two spheres close together generate yet another mode structure. Therefore negative ϵ gives rise to a rich variety of electromagnetic structure decorating the surfaces of metals with a complexity controlled by the geometry of the surface. In fact, the electromagnetic response of metals in the visible region and near ultraviolet is dominated by the negative epsilon concept. Ritchie and Howie [5], Howie and Walsh [2] and many other researchers have shown how important the concept of the plasmon is in the response of metals to incident charged particles.

CHAPTER 2

PERMITTIVITY TENSORS AND HYPERBOLIC METAMATERIAL

2.1 Permittivity Tensors

The electric permittivity ϵ is a constant for a linear, homogeneous and isotropic material; it is a function of space in inhomogeneous media. However, in linear anisotropic media, ϵ is no longer a scalar, but it is a tensor (of rank 2). For an electrically anisotropic medium-

$$\widehat{\epsilon} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yz} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} \tag{2.1}$$

Crystals are, in general, described by a symmetric permittivity tensor for which there always exists a coordinate transformation that transforms the symmetric matrix (tensor) $\widehat{\epsilon}$ to a diagonal matrix-

$$\widehat{\epsilon} = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} \tag{2.2}$$

The three coordinate axes are called the principal axes. For a uniaxial medium two of the three ϵ are equal, that is $\epsilon_{xx} = \epsilon_{yy} = \epsilon$,

$$\widehat{\epsilon} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} \tag{2.3}$$

A highly anisotropic medium having hyperbolic dispersion [8] determined by their effective electric and or magnetic tensors falls into the category of one class of metamaterials called hyperbolic metametaterials. Such metamaterials display anisotropic behavior of uniaxial crystals, and one of the principal components of either their permittivity (ϵ) or permeability (μ) tensors is opposite in sign to the other two principal components [1, 3, 6].

$$\widehat{\epsilon} = \begin{pmatrix} \epsilon_{\perp} & 0 & 0 \\ 0 & \epsilon_{\perp} & 0 \\ 0 & 0 & \epsilon_{||} \end{pmatrix} and\ \widehat{\mu} = \begin{pmatrix} \mu_{\perp} & 0 & 0 \\ 0 & \mu_{\perp} & 0 \\ 0 & 0 & \mu_{||} \end{pmatrix} \tag{2.4}$$

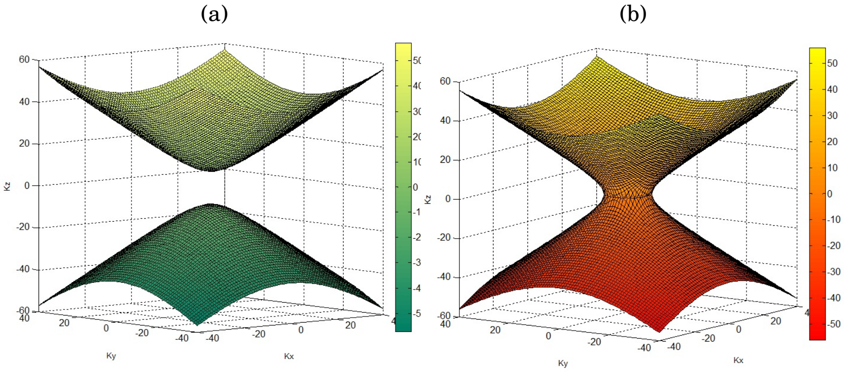
Here the two subscripts ,Äñ and $\perp$ subscripts denote the parallel and perpendicular components of the anisotropy axis, respectively.

2.2 Hyperbolic Metamaterial

In our project we focus our attention on electric hyperbolic materials having $\mu_{\perp} = \mu_{||} > 0$ and either $\epsilon_{\perp} > 0$ and $\epsilon_{||} < 0$ or $\epsilon_{\perp} < 0$ and $\epsilon_{||} > 0$. They have unusual properties because of the isofrequency surface of extraordinary waves (Figure 2.1) given by

$$\frac{k_x^2 + k_y^2}{\epsilon_{||}} + \frac{k_z^2}{\epsilon_{\perp}} = \left(\frac{\omega}{c}\right)^2 \tag{2.5}$$

We have simulated equation 2.5 using MATLAB which is depicted in figure 2.1 giving a clear insight to the nature of isofrequency surfaces.


(a)

(b)

FIGURE 2.1. Isofrequency surfaces of extraordinary waves in hyperbolic Metamaterials. Isofrequency surfaces ($\omega(k) = constant$) given by $\mu_{\perp} = \mu_{||} > 0$ and (a) $\epsilon_{\perp} > 0$ and $\epsilon_{||} < 0$ (b) $\epsilon_{\perp} < 0$ and $\epsilon_{||} > 0$.

The reason why hyperbolic metamaterials have become an active research area in recent years is that it is significantly easier to create hyperbolic structures than negative-index metamaterials as the one and only requirement for producing hyperbolic structure is to confine the motion of free electrons in one or two spatial directions. In 1969, the existence of these kind of structures were first experimentally demonstrated in a magnetized plasma [**?**].

CHAPTER 3

HYPERBOLIC METAMATERIAL: EXPECTED BEHAVIORS

In this chapter we present some expected behaviors of hyperbolic metamaterials.

3.1 Hyperbolic Material Behaviors

We have already discussed the isofrequency surfaces (Figure 2.1) of hyperbolic metamaterials. Potemkin [**?**] revisited the problem of the electromagnetic Green function for homogeneous hyperbolic media, where longitudinal and transverse components of the dielectric permittivity tensor have different signs. They analyzed the dipole emission patterns for both dipole orientations with respect to the symmetry axis and for different signs of dielectric constants, and showed that the emission pattern is highly anisotropic and has a characteristic cross-like shape (Figure 3.1) where the waves are propagating within a certain cone and are evanescent outside this cone.

In their theoretical study [**?**] modelled Green function in a hyperbolic metamaterial as a discrete dipole lattice and also discovered that their calculated Green function has a characteristic cross-like shape, spatially modulated due to structure discreteness (Figure 3.2 (a)).

In a different demonstration Chshelokova [**?**] illustrated how to realize an indfinite media with hyperbolic isofrequency surfaces in wavevector space by employing two-dimensional metamaterial transmission lines. They classified different types of such media, and visualized the peculiar character of wave propagation by study of the cross-like emission pattern of a current source placed in the lattice center (Figure 3.2 (b)).

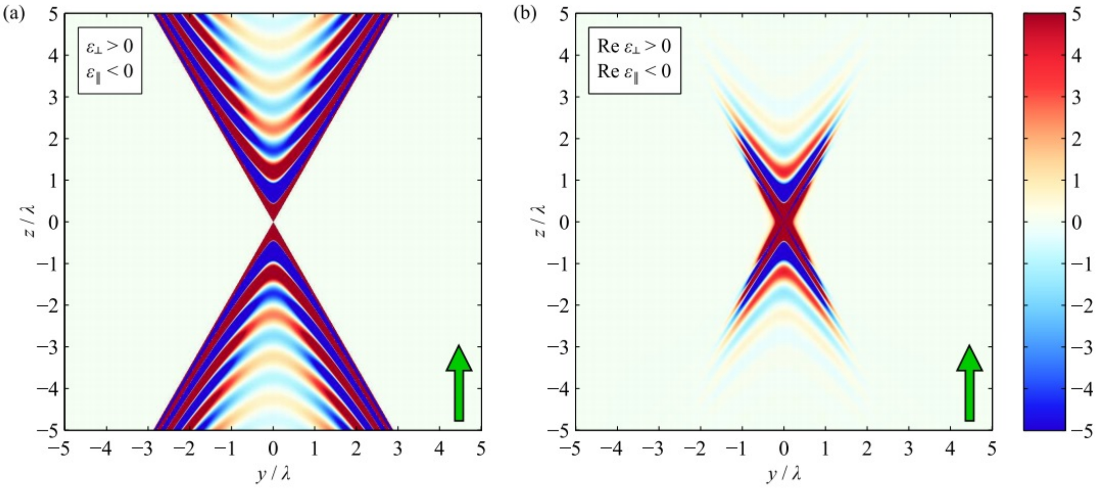


FIGURE 3.1. Dipole emitted electric field pattern in hyperbolic metamaterial for (a) $\epsilon_\perp = 1$ and $\epsilon_{||} = -3$ (b) $\epsilon_\perp = 1 + 0.2i$ and $\epsilon_{||} = -3 + 0.2i$. Dipole moment is parallel to the axis of anisotropy z. [**?**]

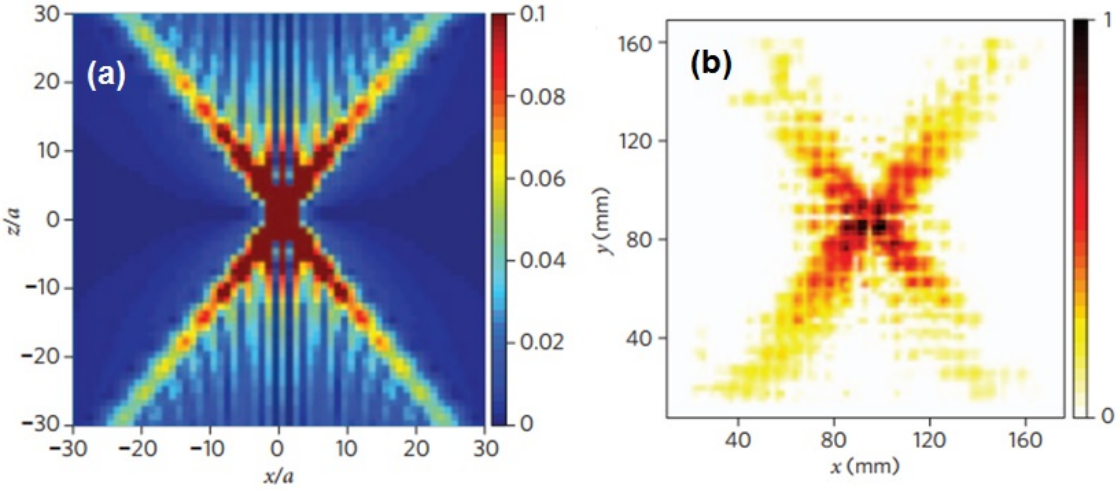


FIGURE 3.2. (a) Green function in a hyperbolic metamaterial modelled as a discrete dipole lattice [**?**]. Here, a is the lattice constant and each point corresponds to the absolute value of the discrete dipole polarization induced by the external point dipole source. (b) Experimental magnetic-field pattern in a two-dimensional transmission line that functions as a hyperbolic metamaterial [**?**].

CHAPTER 4

OUR PROPOSED STRUCTURE

In this chapter we will talk about the properties of set of infinite wires and their permittivity. Eventually a structure will be proposed to realize hyperbolic metamaterial.

4.1 Infinite wire structure

For a set of infinite wires oriented along the Y-axis (Figure 4.1)permittivity is given by [**?**]-

$$\epsilon_{yy} = 1 - \frac{\omega_p^2}{\omega^2} \tag{4.1}$$

which can be expressed in terms of the structural parameters as follows-

$$\omega_p = \frac{2\pi c_o^2}{a^2 \ln \dfrac{a}{r}} \tag{4.2}$$

where velocity of light, c_0 periodic distance, a and radius of the wire, r. We have simulated this equation (Figure 4.2) using MATLAB for structural parameters $a = 20mm$ and $r = 3mm$. We obtained plasma frequency $4.345GHz$. From the figure we also observe that below plasma frequency permittivity is negative.

In light of our calculation for obtaining negative permittivity we propose the same structure along the x-axis as well. As a result of which we will obtain the condition $\epsilon_\perp = \epsilon_{xx} = \epsilon_{yy} = 1 - \frac{\omega_p^2}{\omega^2} < 0$ below the plasma frequency.

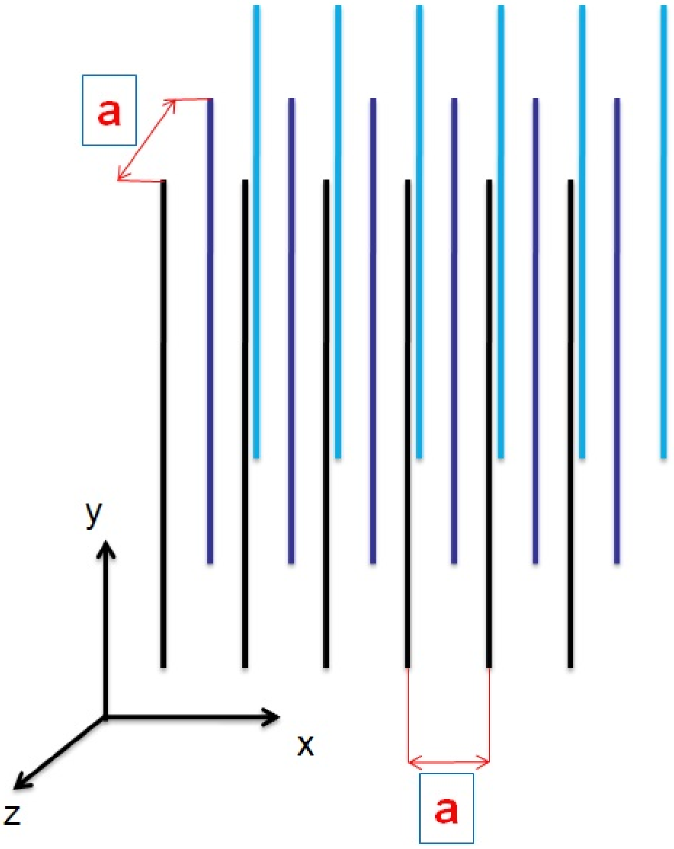


FIGURE 4.1. Set of infinite wires.

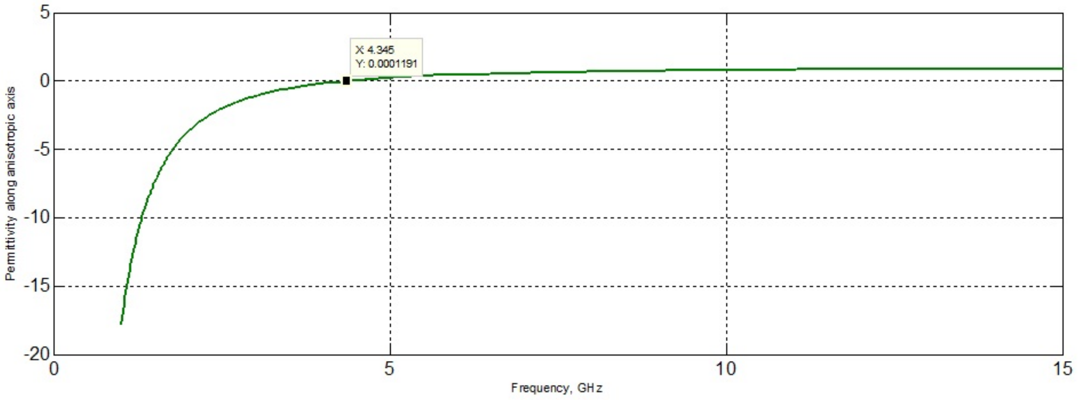


FIGURE 4.2. Variation of permittivity as a function of frequency.

4.2 Stack 2D of Wires Sheets

According to our proposal until now, we have achieved negative permittivity along x and y axis (below the plasma frequency). What remains to obtain hyperbolic material is to realize positive permittivity along z-axis. To achieve that we proposed stacking of wire structure as presented in the figure 4.3.

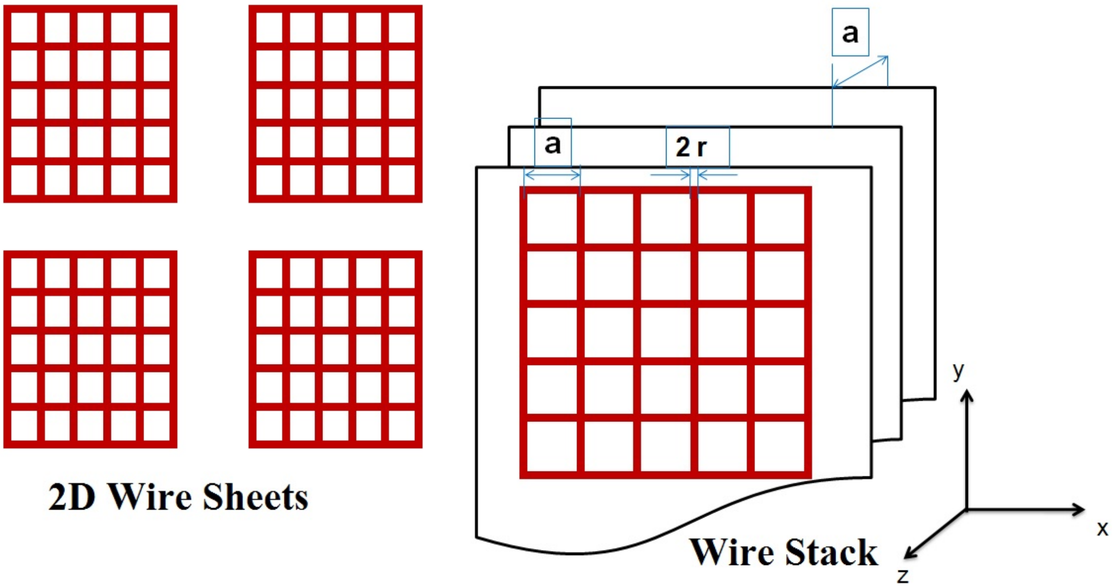


FIGURE 4.3. Our proposed structure.

4.3 Realization of the proposed structure

First we have put copper wires on papers to obtain 2D wire sheets. Then put them on stack with the help of polystyrene frame (polystyrene: acts as vacuum for incident field). Our realized structure is presented in figure 4.4. Now we expect that we have reached the condition for hyperbolic metamaterial (opposite signs permittivity components $\epsilon_{\perp} < 0$ and $\epsilon_{||} > 0$).

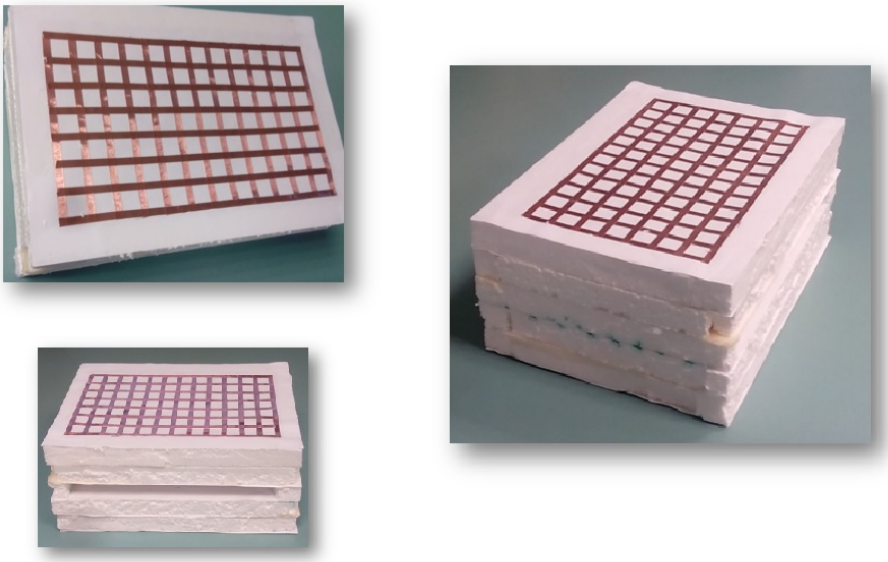


FIGURE 4.4. After realizing our proposed structure.

CHAPTER 5

EXPERIMENTAL INVESTIGATION ON HYPERBOLIC METAMATERIALS

After rigorous study on the basic properties of hyperbolic metamaterial we built a simple structure as proposed in the previous chapter. Then we did measurement on that structure to experimentally verify our claim to realize a hyperbolic metamaterial.

5.1 Experimental Setup

During our experiment we have used a network analyzer which has two independent ports (Port 1 and Port 2). It can simultaneously measure transmission and reflection coefficients (S-Parameters). Port 1 is used to create a small dipole and the Port 2 was connected to a probe. This probe has a small circular current loop which can scan the magnetic field of a given surface. Our experimental setup is shown in Figure 5.1. During our experiment we put the dipole in the middle of our structure and scanned for magnetic field on the $x-y$ plane. Starting from one center-end of the structure we scanned from $0to17cm$ for 100 points along x-axis and $-11to11cm$ for 100 points along y-axis. Frequency was sweeping range was $3-21GHz$.

5.2 Outcome of the experiment

After the experimentation we obtained spatial distribution of the magnetic field intensity. If observed carefully, a crossed pattern is visible. Although this crossed pattern is not demonstrating perfect symmetry but yet this could be inferred from the result that we have successfully realized a hyperbolic metamaterial structure.

FIGURE 5.1. Experimental Setup at the lab. Network analyzer is shown on inset at the top left corner.

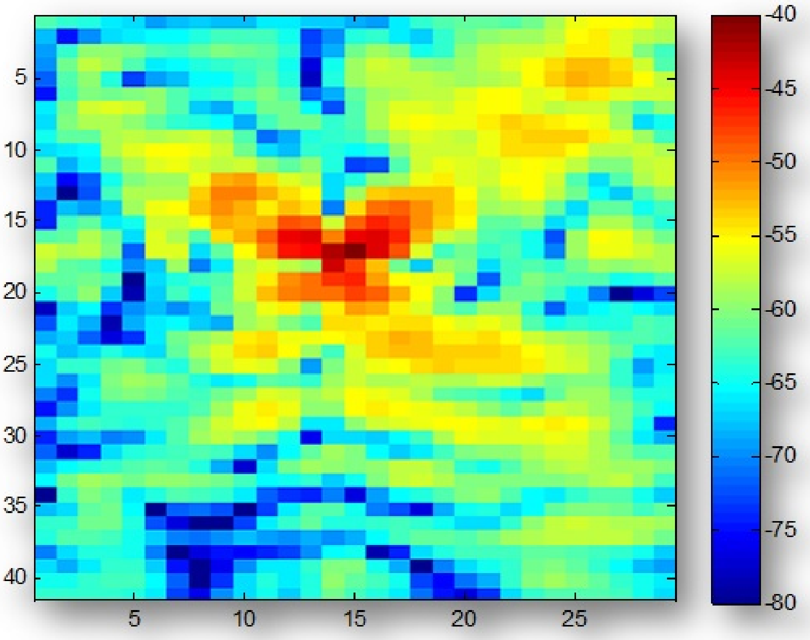


FIGURE 5.2. Measured spatial distribution of the magnetic field intensity.

The disruption in the pattern is mainly attributed to the imperfection of alignment of our built structure. All the six stacks that we have put one over another, were not perfectly aligned to match the structural symmetry.

CHAPTER 6

CONCLUSION

At the end of this project we have achieved understanding on hyperbolic metamaterials. We have successfully demonstrated through experimentation that our proposed structure acts as a hyperbolic metamaterial. Due to the time constraint we could not run more extensive study on the behaviors of our structure. Hyperbolic metamaterials have wide spread application in optical domain. This simple structure can conveniently be realized for optical domain, hence there is huge scope of further study on this structure.

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